Mesh Quality Investigation to Improve Convergence Property of ICCG Method for Finite Element Method

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Abstract — In finite element analysis, it is commonly expected to obtain an accurate result with short time. The computation time is consumed dominantly in solving a system of linear equations. Therefore, in order to solve it with short time, an optimal acceleration factor of ICCG method and a smoothing process to improve the convergence property of the ICCG method are expected. In this paper, we have investigated a correlation of mesh quality and computation time in order to develop a new smoothing method.

I. INTRODUCTION

To perform finite element analysis (FEA), an analysis domain has to be divided into finite elements. In order to obtain an accurate result of the FEA with short time, the finite element mesh with high quality is required. Commonly, the mesh with poor quality yields a system of linear equations with ill-posed condition, then the convergence property of the ICCG method consequently deteriorates, that is, the finite element analysis with poor-quality mesh consumes a lot of time.

It is well known that by using an acceleration factor of the ICCG method [1]-[2], the convergence property can be improved. To speed up the finite element analysis, an optimal acceleration factor, which yields the minimum number of the ICCG iterations, is expected. Therefore, an automatic acceleration factor decision method to estimate it was proposed in [2].

In this paper, we have employed a simple test problem and investigated the usefulness of the automatic acceleration factor decision method proposed in [2]. Then we have defined some functions to evaluate tetrahedral element shapes and investigated the convergence property of the ICCG method with the optimal acceleration factor and the automatically decided one. The presented results help to develop a new smoothing method.

II. MESH EVALUATION METHODS

A. Aspect Ratio Evaluation

Aspect ratio evaluation value λ is defined as the average ratio of the radii of the inscribed sphere and the circumscribed sphere of a tetrahedral element, as follows:

$$\lambda = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{3}\frac{R_i}{r_i}, \tag{1}$$

where N is the number of elements, r_i is the radius of the inscribed sphere and R_i is the radius of the circumscribed sphere, as shown in Fig. 1. When λ is equal to 1.0, the element shape is regular, and the larger λ indicates that the elements are distorted with poor quality.

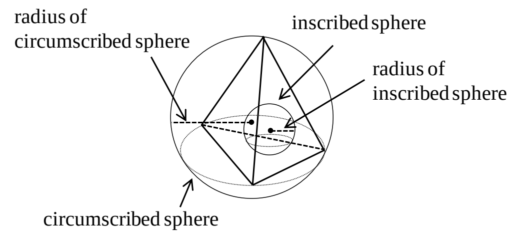


Fig. 1. Concept of the aspect ratio evaluation.

B. Angle Ratio Evaluation

Angle ratio evaluation value φ is defined as the average ratio of the minimum among the angles of two neighbor surfaces on an element and the angle at the regular tetrahedron, as follows:

$$\varphi = \frac{1}{N}\sum_{i=1}^{N}\frac{\theta_i}{\theta_{\mathrm{regular}}}, \tag{2}$$

where θ_i is the minimum of the angles of two neighbor surfaces on the element, as shown in Fig. 2, and $\theta_{\mathrm{regular}}$ is the angle of the regular tetrahedron. When φ is equal to 1.0, the elements are regular, and the smaller φ indicated that the elements are distorted with poor quality.

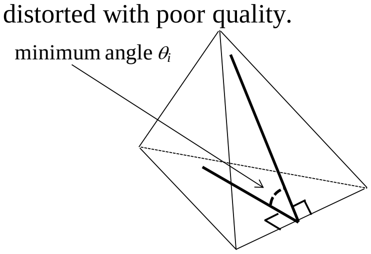


Fig. 2. Concept of the angle ratio evaluation.

C. Volume Ratio Evaluation

Volume ratio evaluation value ε is defined as the average ratio of the volume of the evaluated element and the one of the regular tetrahedron whose edges are as long as the longest edge of the evaluated element (called criterial regular tetrahedron), as follows:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \ , \varepsilon_i = \sqrt[3]{\frac{V_i}{V_{i\mathrm{regular}}}} \ , V_{i\mathrm{regular}} = \frac{\sqrt{2}}{12}h_i^{\ 3}, \tag{3}$$

where V_i is the volume of the evaluated element, $V_{i\mathrm{regular}}$ is the volume of the criterial regular tetrahedron and h_i is the longest edge of the evaluated element, as shown in Fig. 3. When ε is equal to 1.0, the elements are regular, and the smaller ε indicates that the elements are distorted with poor quality.

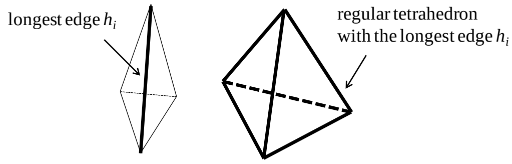


Fig. 3. Concept of the volume ratio evaluation.

III. TEST MODEL

The simple test model consists of a permanent magnet (1.0 T), as shown in Fig. 4. The analysis region is firstly divided into regular hexahedra, and then each hexahedron is divided into six tetrahedra (MESH (W)). Table I shows the properties of MESH (W). For the investigation of the convergence property of the ICCG method, distorted meshes (MESH (X) and the MESH (Z)) are generated by zigzag moving the nodes of MESH (W) into *x*- and *z*-directions, respectively, as shown in Figs. 5(b) and (c). The moving distance of nodes is varied from 1.00 to 2.49 mm.

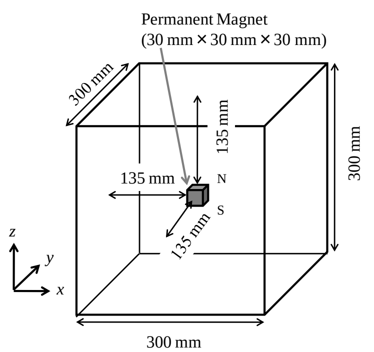
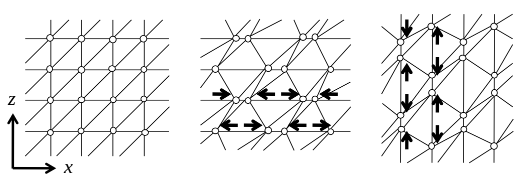


Fig. 4. Test model consisting of a permanent magnet.

TABLE I
MESH PROPERTIES

Nodes	Elements	Edges of unknown
226,981	1,296,000	1,479,780



(a) MESH (W) (b) MESH (X) (c) MESH (Z)

Fig. 5. Conceptual illustration of meshes.

IV. INVESTIGATION OF CONVERGENCE PROPERTY

Figs. 6 and 7 show the number of ICCG iterations with the optimal acceleration factor γ_{opt} and the automatically decided acceleration factor γ_{auto} versus the evaluation values λ, φ and ε, respectively. The optimal acceleration factor γ_{opt} was found by solving many times with the different acceleration factors.

Fig. 6(a) shows that the number of ICCG iterations with γ_{auto} is larger than that with γ_{opt} as the quality of the mesh deteriorates. Furthermore, a few trials with γ_{auto} took more than 3000 iterations. Therefore, two problems have to be addressed. One is to develop the more accurate automatic acceleration factor decision method. Second is to improve the quality of the mesh. The more accurate automatic acceleration factor decision is presented in the paper submitted to this conference, the COMPUMAG 2011 [3]. This paper mainly describes on the investigation of the mesh quality for developing the smoothing method.

As seen in Figs. 6 and 7, a similar correlation is found in each evaluation. The results of the angle evaluation (Figs. 6(b) and 7(b)) show almost the same trend. When the angle ratio evaluation φ is smaller than 0.4, the convergence property deteriorates drastically. The aspect ratio evaluation is commonly employed, but the mesh quality should be evaluated by the angle ratio evaluation φ. In addition, a new smoothing method and its detailed procedure will be proposed to improve the angle ratio evaluation φ in the full paper.

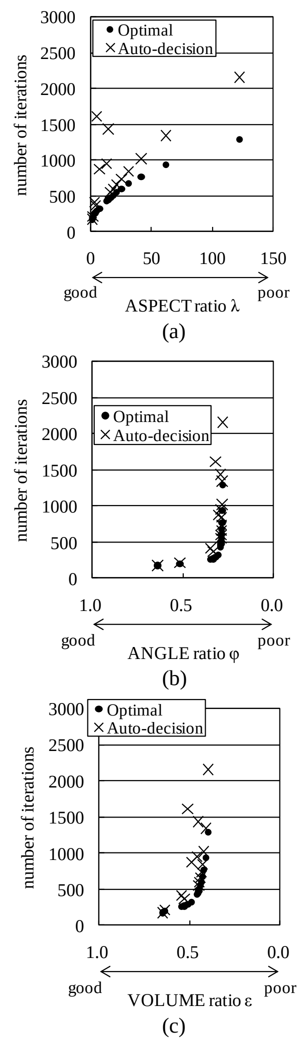


Fig. 6. Evaluation of mesh quality of MESH (X).

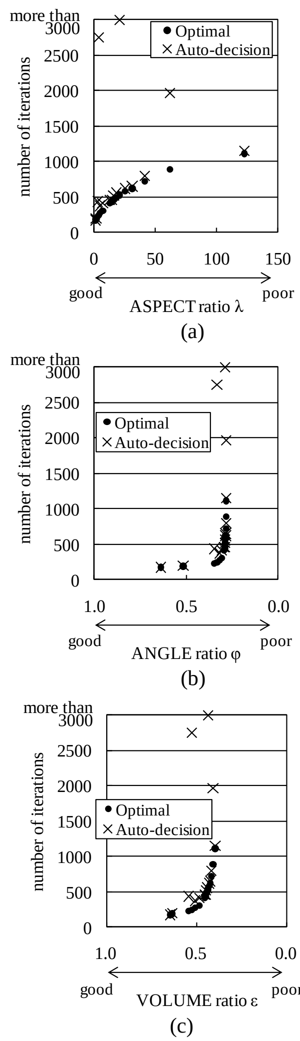


Fig. 7. Evaluation of mesh quality of MESH (Z).

V. REFERENCES

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[2] K. Fujiwara, T. Nakata and H. Fusayasu, "Acceleration of Convergence Characteristic of the ICCG Method," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1958-1961, 1993.

[3] A. Takada, S. Noguchi and H. Igarashi, "A New Acceleration Factor Decision Method for ICCG Method Utilizing Quality of Finite Element Mesh," submitted to the COMPUMAG 2011.